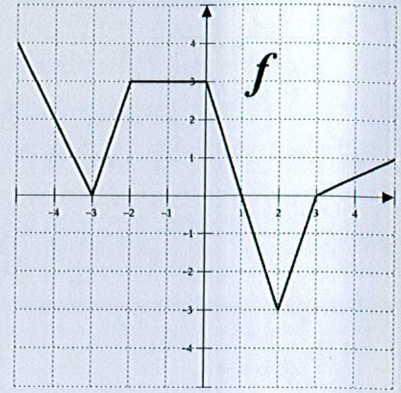
Let $g(x) = \int_2^x f(t)\, dt$, where f is the function whose graph is shown on the right. **SCORE: _____ / 30 PTS**



[a] Find $g(-5)$.

$$\int_2^{-5} f(t)\,dt = -\int_{-5}^{2} f(t)\,dt$$

$$= -\left[\tfrac{1}{2}(4)(2) + \tfrac{1}{2}(2+4)(3) - \tfrac{1}{2}(3)(1)\right]$$

$$= -11\tfrac{1}{2}$$

2 EACH

[b] Find $g'(-1)$. **Explain your answer very briefly.**

$g'(-1) = f(-1) = 3$ 2½

[c] Find all intervals over which g is increasing and concave down. **Explain your answer very briefly.**

$g'(x) = f(x) > 0$ AND DECREASING ON $(-5,-3)$ AND $(0,1)$

2½ EACH

[d] Find all critical numbers of g that do **NOT** correspond to local maxima nor local minima. **Explain your answer very briefly.**

$g'(x) = f(x) = 0$ BUT DOESN'T CHANGE SIGNS @ $x = -3$

2½ EACH

If $g(x) = \int_{2x}^{x^2} \ln(1+t^3)\,dt$, find $g''(1)$.

SCORE: _____ / 15 PTS

$$g(x) = \int_{2x}^{0} \ln(1+t^3)\,dt + \int_{0}^{x^2} \ln(1+t^3)\,dt = \int_{0}^{x^2} \ln(1+t^3)\,dt - \int_{0}^{2x} \ln(1+t^3)\,dt$$

$$g'(x) = \ln(1+(x^2)^3)\cdot 2x - \ln(1+(2x)^3)\cdot 2 = 2x\ln(1+x^6) - 2\ln(1+8x^3)$$

$$g''(x) = 2\ln(1+x^6) + \frac{2x\cdot 6x^5}{1+x^6} - \frac{2\cdot 24x^2}{1+8x^3}$$

1

$$g''(1) = 2\ln 2 + \frac{12}{2} - \frac{48}{9}$$

$$= 2\ln 2 + \frac{2}{3}$$

2 EACH EXCEPT AS INDICATED

$$\int_{-2}^{2} \left(3\sqrt{16-(x-2)^2} - 2x^2 \sin x\right) dx$$ 1

$= 3\int_{-2}^{2} \sqrt{16-(x-2)^2}\, dx - \int_{-2}^{2} 2x^2 \sin x \, dx$ 2

$y = \sqrt{16-(x-2)^2}$

$(x-2)^2 + y^2 = 16$

CIRCLE RADIUS 4 2

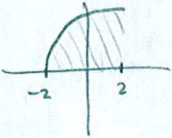
CENTER $(2,0)$

$2(-x)^2 \sin(-x)$

$= -2x^2 \sin x$ 2

2 ODD, CONTINUOUS 2

INTEGRAL $= 0$ 2



$= 3 \cdot \frac{1}{4}\pi(4)^2$ 3

$= 12\pi$ 2

$$\int \frac{14\sin 8y - 21e^{2y}}{\sqrt[3]{6e^{2y} + \cos 8y}}\,dy$$

4

$$\underline{u = 6e^{2y} + \cos 8y}$$

$$du = (12e^{2y} - 8\sin 8y)\,dy$$

$$= 4(3e^{2y} - 2\sin 8y)\,dy$$

$$-\frac{7}{4}du = -7(3e^{2y} - 2\sin 8y)\,dy$$

$$= (14\sin 8y - 21e^{2y})\,dy$$

$$= \underbrace{-\frac{7}{4}}_{4}\ \underbrace{\int \frac{1}{\sqrt[3]{u}}\,du}_{3}$$

$$= -\frac{7}{4}\underbrace{\left(\frac{3}{2}\right)u^{\frac{2}{3}}}_{4} + C$$

$$= \underbrace{-\frac{21}{8}(6e^{2y} + \cos 8y)^{\frac{2}{3}}}_{3} + \underbrace{C}_{2}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \theta^2 \csc\theta \, d\theta$$

$\theta^2 \csc\theta$ IS DISCONTINUOUS

@ $\theta = 0$

3

SO FTC DOESN'T APPLY

2

d $\int (3-4t)\sqrt{2t-5}\,dt$

4

$u = 2t-5$

$du = 2\,dt$

$\frac{1}{2}\,du = dt$

$t = \frac{u+5}{2}$

$3-4t = 3-4\left(\frac{u+5}{2}\right)$

$= -2u-7$

2

$= \frac{1}{2}\int (-2u-7)\sqrt{u}\,du$ 5

$= \int \left(-u^{\frac{3}{2}} - \frac{7}{2}u^{\frac{1}{2}}\right) du$

$= -\frac{2}{5}u^{\frac{5}{2}} - \frac{7}{2}\left(\frac{2}{3}\right)u^{\frac{3}{2}} + C$ 4

$= \underbrace{-\frac{2}{5}(2t-5)^{\frac{5}{2}} - \frac{7}{3}(2t-5)^{\frac{3}{2}}}_{3} + \underbrace{C}_{2}$

The table gives the acceleration of a car (in meters/minute^2) at various times (in minutes). **SCORE: _____ / 15 PTS**
At time $t = 3$, the velocity of the car was 5 meters/minute.

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$a(t)$ ~~$v(t)$~~	3	8	6	5	2	4	7	9	10	13	11	12	15	14

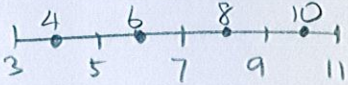
[a] Write an expression involving an integral for the velocity of the car at $t = 11$.

$$v(11) - v(3) = \int_3^{11} a(t)\,dt$$

$$v(11) = \underbrace{5}_{2} + \underbrace{\int_3^{11} a(t)\,dt}_{5}$$

[b] Estimate the velocity of the car at $t = 11$ using [a], 4 subintervals and midpoints. Specify the units of your answer.

$$\Delta t = \frac{11-3}{4} = 2$$



$$5 + (a(4) + a(6) + a(8) + a(10))\Delta t = 5 + \underbrace{(2+7+10+11)}_{4}\underbrace{(2)}_{2}$$

$$1 = \underbrace{65}\ \underbrace{m/min}\ 1$$

Find $f(t)$ and a such that $1+\int_a^x \ln(f(t))\,dt = \frac{2}{x^3}$. SCORE: _____ / 15 PTS

$\frac{d}{dx}\left(1+\int_a^x \ln f(t)\,dt\right) = \frac{d}{dx}\left(\frac{2}{x^3}\right)$

$\ln f(x) = -6x^{-4}$ 5

$f(x) = e^{-6x^{-4}}$

$f(t) = e^{-6t^{-4}}$ 2

$1+\int_a^x \ln e^{-6t^{-4}}\,dt = \frac{2}{x^3}$

$1+\int_a^x -6t^{-4}\,dt = \frac{2}{x^3}$

$1+2t^{-3}\Big|_a^x = \frac{2}{x^3}$

$1+\cancel{2x^{-3}}-2a^{-3} = \cancel{\frac{2}{x^3}}$ 5

$\frac{1}{2} = a^{-3}$

$a = \sqrt[3]{2}$ 3